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The Valuation of Convertible Notes Using a Binomial Model

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The most widely used mathematical models for the valuation of convertible notes are modified binomial models and the Monte Carlo simulation model. While the Monte Carlo simulation model is the more robust of the two, a modified binomial model may be the more appropriate model for valuing convertible notes in certain circumstances.

A number of valuators cite the proprietary nature of their binomial models as a rational for not sharing their models.

Standards Rule 10-1 (b) of the 2020-2021 Uniform Standards of Professional Practice (USPAP) indicates that an appraisal report must have sufficient information to enable the intended user(s) of the appraisal to understand the report properly.

Convertible Notes

Convertible notes are a type of debt instrument that give the noteholder the option of converting the note into a number shares of common stock in the debtor's company. Thus, convertible notes have both fixed income and equity features that make them complicated to value.

We have seen numerous attempts by valuators to employ the binomial model without a thorough understanding of the complexities of the model or, if they do understand the complexities of the model, make undisclosed adjustments to the model and leave it to the reader to figure out the modification(s) and, as John Lennon quipped about understanding the meaning of the lyrics of "I Am the Walrus," "Let the ******* work that one out."[1]

Our own online research and search of our library revealed that there are numerous articles, chapters, and models with bad math, bad equations, notational errors, or undisclosed rounding in some equations.

The most commonly cited version of the binomial model is the one modified by Goldman Sachs. We will present a version of the Goldman Sachs binomial model below.[2]

A binomial model normally has two trees:

1. A stock price tree; and
2. A corresponding option tree.

For convertible notes, the binomial model has four trees:

1. A stock price tree;
2. A conversion probability tree;
3. A credit-adjusted spread tree; and

1 beatlesbible.com/songs/i-am-the-walrus/2.

2 Filippo Stefanini, *Handbook of Finance, Volume III, Valuation, Financial Modeling, and Quantitative Tools,* 2008, Chapter 41, "Quantitative Models to Value Convertible Bonds," pages 445-449. This model, as published, contains a notational error (did not include a "-" in front of the risk-free rate).

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4. A convertible note value tree.

The binomial model employs discrete mathematics, which result in an approximate conclusion where the more steps employed, the more precise the answer. We have found that a common error of the binomial model is to assume the term of the instrument dictates the number of steps. Instead, the number of steps should be increased in the model until the results no longer materially change.

For the purposes of this article, we will be valuing a convertible note with the following terms:

Maturity	= 5 years
Steps	= 5
Risk-free rate	= 4%
Credit spread	= 2%
Dividend yield	= 0%
Face value of note	= $100.00
Coupon rate	= 10%
Coupon payments per year	= 1
Conversion ratio	= 100%
Stock price	= $85.00
Stock volatility	= 10%

Stock Price Tree

The stock price tree for valuing a convertible note is the same as for valuing a stock option.

The first step in the binomial option-pricing model is an intermediate set of equations for determining the expected up and down movement of the underlying stock price at each step.

$$U = e^{\sigma\sqrt{step}}$$

$$D = \frac{1}{U}$$

Where:

U = The up factor, or the factor by which the stock will rise in any given time step that the stock's volatility determines.

e = 2.71828, the base of the natural log function.[3]

δ = Standard deviation of the annualized continuously compounded rate of return on the underlying stock.

$step$ = Time steps. The time (in years) until expiration divided by the total number of steps, e.g., if an option has five years and six months remaining until expiration and we are analyzing the option using a 100-step model, step would equal 5.5/100, or 0.055. In our example, it would be 5/5 = 1.

D = The down factor, or the factor by which the stock will decline in any given time step. It is the reciprocal of the up factor.

3 $e = \lim_{n\to\infty}(1+X)^{1/X} = 2.71828$

Applying the formula results in the following:

$$U = e^{10\%\sqrt{5/5}}$$

$$U = 1.1052$$

$$D = \frac{1}{1.1052}$$

$$D = 0.9048$$

Starting with the current share price and solving for each possible outcome using the up (U) and down (D) factors, one can construct the lattice of the future stock prices.

The resulting five-step stock tree is shown in Exhibit 1.

Conversion Probability Tree

To determine the conversion probability, one needs to perform an intermediate calculation to estimate the risk-neutral probability of an up movement in the stock price and the risk-neutral probability of a down movement in the stock price.

Exhibit 1. Five-Step Stock Tree

Years		1.00	2.00	3.00	4.00	5.00
Steps		1	2	3	4	5
						$140.14
					$126.81	
				$114.74		$114.74
			$103.82		$103.82	
		$93.94		$93.94		$93.94
Stock Price Lattice	$85.00		$85.00		$85.00	
		$76.91		$76.91		$76.91
			$69.59		$69.59	
				$62.97		$62.97
					$56.98	
						$51.56

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The formula is:

$$P = \frac{e^{(r-div)(step)} - D}{U - D}$$

Where:

P = the risk-neutral up probability.

e = 2.71828, the base of the natural log function.

r = the risk-free interest rate (annualized and continuously compounded) having the same maturity as the convertible note.

div = the annualized dividend yield.

$Step$ = time steps (time/steps).

U = the up factor.

D = the down factor.

The risk-neutral down probability is 1 - P.

Applying the formula we get the following:

$$P = \frac{e^{(4\%-0\%)(1/5)} - 0.9048}{1.1052 - 0.9048}$$

$$P = 67.87\%$$

Therefore, the risk-neutral down probability is 32.13% (1 - 67.87%).

Now we can construct the conversion probability tree. The tree begins with the ending stock prices. If the ending stock price times the conversion ratio is greater than the amount the noteholder would receive when the note matures, then the noteholder would elect to convert and enters a 100% in the corresponding node. Otherwise, the noteholder would not convert and collect the principal and the final coupon payment and enters a 0% in the corresponding node.

In this case, the final nodes of the conversion probability tree would be:

100.00%

100.00%

0.00%

0.00%

0.00%

0.00%

The two highest ending stock values of $140.14 and $114.74 were greater than the final bond payment of $110.00. In the remaining four instances, the stock price was less than the final bond payment of $110. Therefore, the probability of conversion in the first two nodes is 100% and the probability of conversion in the remaining four nodes is 0%.

For the remaining nodes, we work from right to left and apply the following equation:

$$q_{n,i} = (q_{n+1,i+1}P) + (q_{n+1,i})(1 - P)$$

Where:

$q_{n,i}$ = the conversion probability of the node being calculated.

$q_{n+1,i+1}$ = the conversion probability of the up node to the right of the node being calculated.

$q_{n+1,i}$ = the conversion probability of the down node to the right of the node being calculated.

In this case, only one node would result in a value other than 100%, or 0%. The calculation for the node is as follows:

$$q_{n,I} = (100\%)\,(67.87) + (0\%)\,(32.13\%)$$

$$q_{n,I} = 67.87\%$$

Working our way from right to left results in the conversion probability lattice shown in Exhibit 2.

Credit-Adjusted Spread Tree

The third tree is used to determine the discount rate. If the convertible note is out of the money (i.e., a plain vanilla note), future cash flows should be discounted at the discount rate appropriate for the plain vanilla debt. Therefore, the rate would be the risk-free rate, plus the appropriate credit spread for the plain vanilla note.

Conversely, if the convertible note is in the money, the future cash flows would be discounted at the risk-free rate since the risk is already reflected in the volatility because the conversion feature is being priced as if it were a stock option.

Again, we build the tree from right to left starting at maturity. Again, the first two nodes in the right-most nodes are in the money, so the appropriate discount rate is the risk-free rate (4%) and the remaining nodes show a spread-adjusted discount rate of 6% (4% + 2%).

The final nodes of the credit-adjusted spread tree would be:

4.00%

4.00%

6.00%

6.00%

6.00%

6.00%

For the remaining nodes, we work from right to left and apply the following equation:

$$r_{n,i} = \left(q_{n,i} r\right) + (1 - q_{n,i})(r + k)$$

Where:

$r_{n,i}$ = the discount rate of the node being calculated.

$q_{n,i}$ = the conversion probability corresponding to the node being calculated.

r = the risk-free rate corresponding to the term of the convertible note.

k = the credit spread of the convertible note.

Exhibit 2. Conversion Probability Lattice

Years		1.00	2.00	3.00	4.00	5.00
Steps		1	2	3	4	5
						100.00%
					100%	
				89.68%		100.00%
			75.67%		67.87%	
		61.40%		46.07%		0.00%
Conversion Probability Lattice	48.50%		31.27%		0.00%	
		21.22%		0.00%		0.00%
			0.00%		0.00%	
				0.00%		0.00%
					0.00%	
						0.00%

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In this case, only one node would result in a value other than 4%, or 6%. The calculation for the node is as follows:

$$r_{n,i} = (67.9\%\ x\ 4\%) + (1 - 67.9\%)(4\% + 2\%)$$

$$r_{n,i} = 4.64\%$$

Working our way from right to left results in the credit-adjusted spread lattice shown in Exhibit 3.

Convertible Note Value Tree

The final tree is used to determine the value of the convertible note. Again, we build the tree from right (maturity) to left (the valuation date).

At maturity, the note will be the greater of the conversion value, or the note value (in this case, the note value is $110.00). The conversion value is the stock price multiplied by the conversion ratio. In our example for the top-most node, the value is $140.14 ($140.14 × 1). Filling in the convertible note values at maturity results in the following:

$140.14

$114.74

$110.00

$110.00

$110.00

$110.00

As we work our way to the left, we apply the following formula to each node:

$$p_{n,i} = \max\left[S_{n,i}m, (Pp_{n+1,i}e^{-r_{n+1,}step}) + ((1 - P)p_{n-1,i}e^{-r_{n-1,}step})\right]$$

Where:

$p_{n,i}$ = value of the convertible note at the node being calculated.

$S_{n,i}$ = value of the stock corresponding to the node being calculated.

m = the conversion ratio.

P = the risk-neutral up probability.

Exhibit 3. Credit-Adjusted Spread Lattice

Years		1.00	2.00	3.00	4.00	5.00
Steps		1	2	3	4	5
						4.00%
					4.00%	
				4.21%		4.00%
			4.49%		4.64%	
		4.77%		5.08%		6.00%
Credit-Adjusted Spread Lattice	5.03%		5.37%		6.00%	
		5.58%		6.00%		6.00%
			6.00%		6.00%	
				6.00%		6.00%
					6.00%	
						6.00%

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$p_{n+1,i}$ = value of the convertible note at the up node to the right of the node being calculated.

e = 2.71828, the base of the natural log function.

$r_{n+1,i}$ = the discount rate at the up node to the right of the node being calculated.

$p_{n-1,i}$ = value of the convertible note at the down node to the right of the node being calculated.

$r_{n-1,i}$ = the discount rate at the down node to the right of the node being calculated.

$step$ = time steps. The time (in years) until expiration divided by the total number of steps.

We will continue to use the same node for our example of the application of the formula. Therefore, the value in our sample node is as follows:

$$p_{n-1,i} = \max\left[\$103.82\ x\ 1, (67.87\%\ x\ \$114.74e^{-4\%\left(\frac{5}{5}\right)}) + ((1-67.87\%)\ x\ \$110.00e^{-6\%\left(\frac{5}{5}\right)})\right]$$

$$p_{n-1,i} = \$108.10$$

Working our way from right to left results in the convertible note lattice shown in Exhibit 4.

A five-step model will rarely result in a correct estimate of the value of a convertible note. Instead, one must test the sensitivity of the concluded value by repeating the process using a significantly greater number of steps. Using the above inputs, a 10-step model results in a value of $104.44, a 100-step model results in a value of $91.38, and a 250-step model results in a value of $91.39.

According to the AICPA Accounting and Valuation Guide, "Valuation of Portfolio Company Investments of Venture Capital and Private Equity Funds and Other Investment Companies," "[n]ote, there are other valuation models for valuing convertible notes that use alternative approaches

Exhibit 4. Convertible Note Lattice

Years		1.00	2.00	3.00	4.00	5.00
Steps		1	2	3	4	5
						$140.14
					$126.81	
				$115.85		$114.74
			$106.35		$108.10	
		$97.92		$101.39		$110.00
Value of Convertible Bond	$90.36		$94.93		$103.59	
		$88.86		$97.56		$110.00
			$91.88		$103.59	
				$97.56		$110.00
					$103.59	
						$110.00

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Since there is virtually no transparent guidance for using binomial models to value convertible notes, we suggest that valuators provide fully functional binomial models when requested so that reviewers can "look under the valuator's hood" and the valuation of these complex instruments can be subject to the same scrutiny that most other valuations are subject to without the reviewers being reminded of John Lennon's quote.

4 See B.10.09, which is part of an overview of valuing convertible instruments that begins on B.10.01. The discussion includes an example of using a binomial model to value a convertible note beginning on B.10.06.

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